

DSC 140A

Probabilistic Modeling & Machine Learning

Lecture 18 | Part 1

Boosting

Today

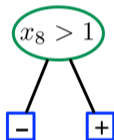
- ▶ Can we **combine** very **simple** models and get good results?
- ▶ **Yes: boosting.**

Weak Learners

- ▶ A **weak classifier** is one which performs only a little better than chance.
- ▶ A learning algorithm capable of consistently producing weak classifiers is called a **weak learner**.
- ▶ Usually very simple, fast.

Example

- ▶ A **decision stump** is a **weak classifier**.



- ▶ **Weak learner:** the strategy discussed last time for picking question.

Example

- ▶ The full decision tree learning algorithm is a **strong learner**.

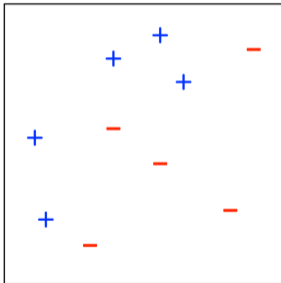
The Question

- ▶ Can we “boost” the quality of a weak learner?

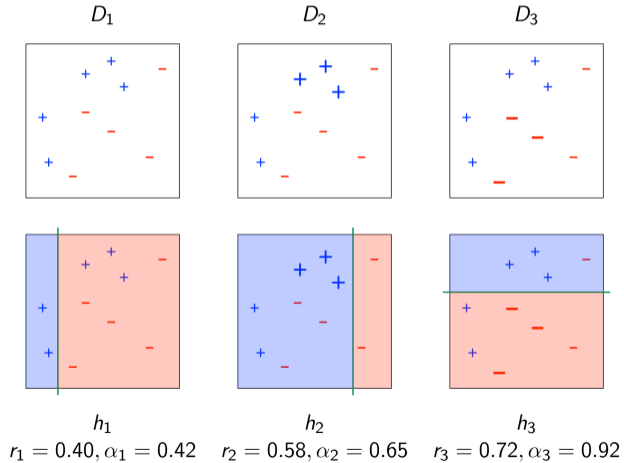
Boosting: The Idea

- ▶ Train a weak classifier, $H_1 : \mathcal{X} \rightarrow [-1, 1]$.
- ▶ Increase weight (importance) of misclassified points, train another classifier H_2 .
- ▶ Repeat, creating more classifiers, updating weights.
- ▶ Final classifier: a linear combination of H_1, \dots, H_k .

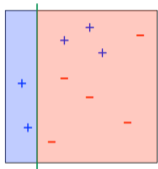
Example



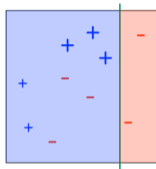
Example



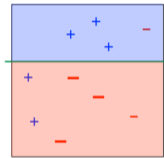
Example



h_1
 $\alpha_1 = 0.42$



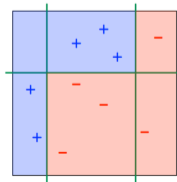
h_2
 $\alpha_2 = 0.65$



h_3
 $\alpha_3 = 0.92$

Final classifier:

$$\text{sign}(0.42h_1(x) + 0.65h_2(x) + 0.92h_3(x))$$



The Details

- ▶ **Q1:** How do we measure the performance of a classifier on a weighted data set?
- ▶ **Q2:** How do we update the point weights?
- ▶ **Q3:** How do we combine the classifiers?

AdaBoost

- ▶ Yoav Freund (UCSD) and Robert Schapire.
- ▶ A theoretically-sound answer to these questions.

Q1: Measuring Performance

- ▶ Suppose weights at step t are in $\vec{\omega}^{(t)}$.
 - ▶ Assume normalized s.t. weights add to one.

- ▶ We use weights to learn a classifier
 $H_t : \mathcal{X} \rightarrow [-1, 1]$.

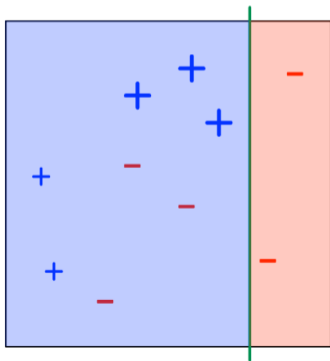
$$y_i H(\vec{x}^{(i)})$$

- ▶ The “margin”:

$$r_t = \sum_{i=1}^n \omega_i^{(t)} \underbrace{y_i H_t(\vec{x}^{(i)})}_{\text{margin}} \in [-1, 1]$$

The Margin

$$r_t = \sum_{i=1}^n \omega_i^{(t)} y_i H_t(\vec{X}^{(i)}) \in [-1, 1]$$



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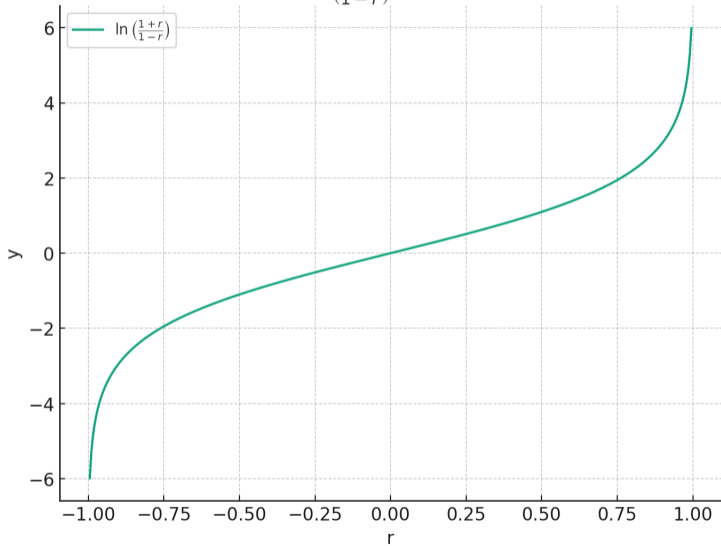
- ▶ The larger r_t , the better H_t is doing on the “important” points.

Q1: Measuring Performance

- ▶ The **performance** of H_t :

$$\alpha_t = \frac{1}{2} \ln \frac{1 + r_t}{1 - r_t}$$

Plot of $\ln\left(\frac{1+r}{1-r}\right)$ for $r \in [-1, 1]$



Q2: Updating Weights

- ▶ We use weights to learn a classifier $H_t : \mathcal{X} \rightarrow [-1, 1]$.
- ▶ Weigh misclassified points more heavily.
- ▶ Point is misclassified if $y_i H_t(\vec{x}^{(i)}) < 0$

Q2: Updating Weights

- ▶ This will do the trick:

$$\omega_i^{(t+1)} \propto \omega_i^{(t)} \cdot \exp(-\alpha_t y_i H_t(\vec{x}^{(i)}))$$

- ▶ α because we normalize.

Q3: Combining Classifiers

- ▶ The final classifier:

$$H_t(\vec{X}) = \sum_{t=1}^T \alpha_t H_t(\vec{X})$$

AdaBoost

Given data $(\vec{x}^{(1)}, y_1), \dots, (\vec{x}^{(n)}, y_n)$, labels in $\{-1, 1\}$.

- ▶ Initialize weight vector, $\vec{\omega}^{(1)} = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^T$
- ▶ Repeat:
 - ▶ Give data and weights $\vec{\omega}^{(t)}$ to weak learner. Receive a classifier, $H_t : \mathcal{X} \rightarrow \{-1, 1\}$ back.
 - ▶ Calculate “performance”, $\alpha_t = \frac{1}{2} \ln \frac{1+r_t}{1-r_t}$
 - ▶ Update $\vec{\omega}^{(t+1)} \propto \omega_i^{(t)} \cdot \exp(-\alpha_t y_i H_t(\vec{x}^{(i)}))$
- ▶ Final classifier: $H_t(\vec{x}) = \sum_{t=1}^T \alpha_t H_t(\vec{x})$

Example: Decision Stumps

- ▶ To learn decision stump, given data and $\vec{\omega}^{(t)}$.
- ▶ Try all features, thresholds.
- ▶ Choose that which maximizes the margin:

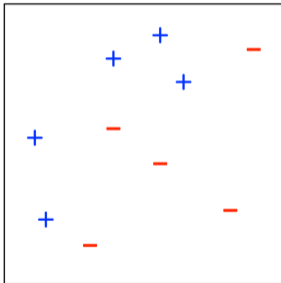
$$r_t = \sum_{i=1}^n \omega_i^{(t)} y_i H_t(\vec{x}^{(i)}) \in [-1, 1]$$

Example: Decision Stumps

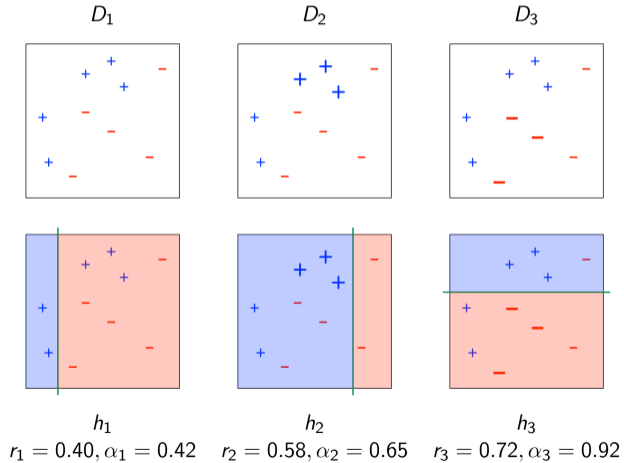
- ▶ To learn decision stump, given data and $\vec{\omega}^{(t)}$.
- ▶ Try all features, thresholds.
- ▶ Equivalently, choose that which maximizes the performance:

$$\alpha_t = \frac{1}{2} \ln \frac{1 + r_t}{1 - r_t}$$

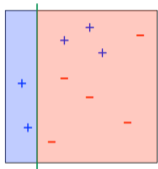
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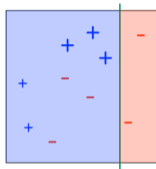
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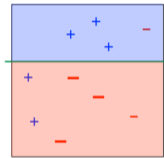
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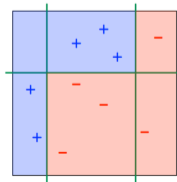
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Final classifier:

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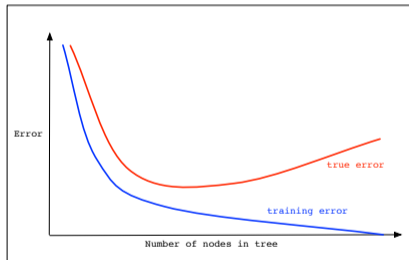


Theory

Suppose that on each round t , the weak learner returns a rule H_t whose error on the step t weighted data is $\leq \frac{1}{2} - \gamma$. Then after T rounds, the training error of the combined rule H is at most $e^{-\gamma^2 T/2}$.

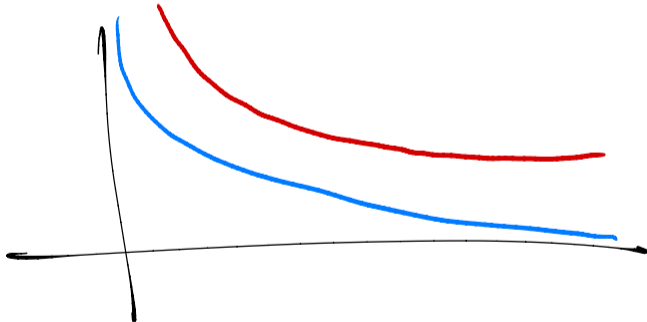
Generalization

- ▶ Boosted decision stumps are really resistant to overfitting.



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Why not?

- ▶ Why use weak learners?
- ▶ What if we replace decision stumps with SVMs or logistic regression?

Why not?

- ▶ Why use weak learners?
- ▶ What if we replace decision stumps with SVMs or logistic regression?
- ▶ You can, but weak learners are **fast** to learn.
- ▶ The point of boosting is that weak learners are “just as good” as strong learners.

DSC 140A

Probabilistic Modeling & Machine Learning

Lecture 18 | Part 2

Random Forests

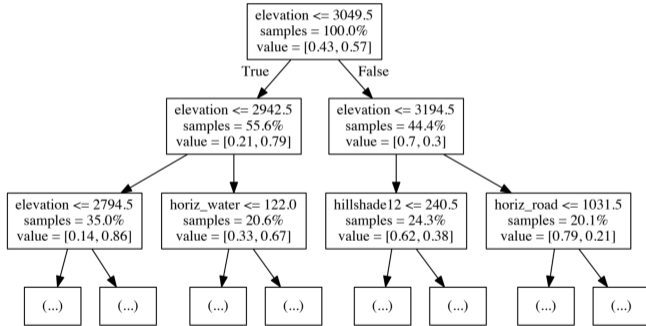
Let's Try

- ▶ Decision trees are susceptible to overfitting.
- ▶ Let's try using boosted decision trees.

Example: Forest Cover Type

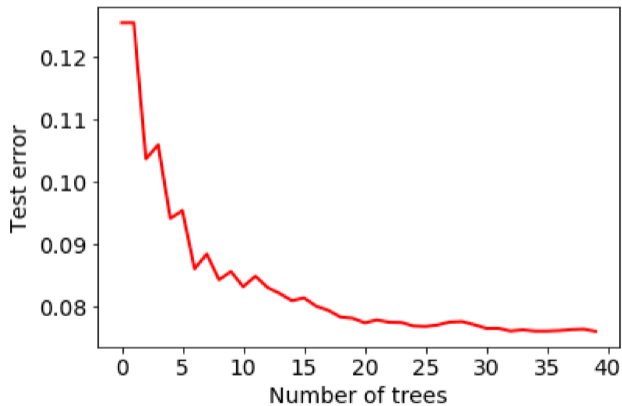
- ▶ **Goal:** predict forest type.
 - ▶ Spruce-fir
 - ▶ Lodgepole pine
 - ▶ etc. 7 classes in total.
- ▶ 54 cartographic/geological features.
 - ▶ Elevation, slope, amount of shade, distance to water, etc.

Decision Tree

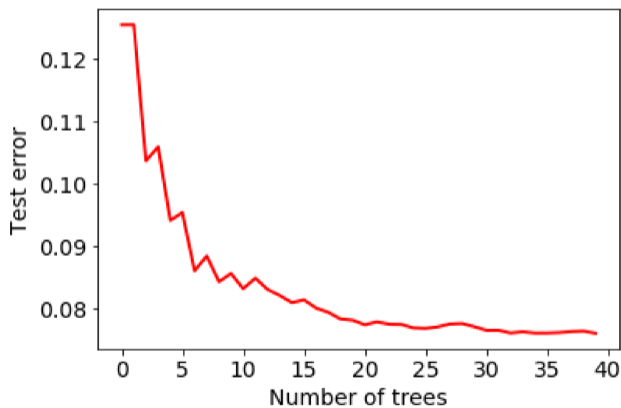


Depth 20. Training error: 1%. Test error: 12.6%.

Boosted Decision Trees



Boosted Decision Trees



Depth 20: Test error: 8.7%. **Slow!**

Another Idea

- ▶ Prevent decision trees from overfitting by “hiding data” randomly.
- ▶ Train a bunch of trees, quickly.
- ▶ Average them to make a final prediction.

Random Forests

- ▶ For $t = 1$ to T
 - ▶ Choose n' training points randomly, with replacement.
 - ▶ Fit a decision tree, H_t .
 - ▶ At each node, restrict to one of k features, chosen randomly.
- ▶ Final classifier: majority vote of H_1, \dots, H_T .
- ▶ Common settings: $n' = n$ (bootstrap), $k = \sqrt{d}$.

Forest Cover Type

- ▶ Decision trees: 12.6% error.
- ▶ Boosted decision trees: 8.7% error (but **slow!**)
- ▶ Random forests: 8.8% error.
 - ▶ 50% of features dropped.
 - ▶ Each individual tree H_1, \dots, H_t has test error around 15%.