DSC 140A - Discussion 06

Problem 1.

In lecture, we defined a kernel function to be a function k which computes the dot product of vectors after they are mapped to some high-dimensional space. The useful thing about kernel functions is that they allow us to compute these dot products without actually mapping vectors them to the high-dimensional space, which can be costly. In this problem, we will consider the the 2nd-order *polynomial kernel*, defined to be

$$k(\vec{x}, \vec{x}') = (1 + \vec{x} \cdot \vec{x}')^2.$$

Let $\vec{\phi}(\vec{x}) : \mathbb{R}^3 \to \mathbb{R}^{10}$ be the mapping:

$$\vec{\phi}(\vec{x}) = (1, x_1^2, x_2^2, x_3^2, \sqrt{2}\,x_1, \sqrt{2}\,x_2, \sqrt{2}\,x_3, \sqrt{2}\,x_1x_2, \sqrt{2}\,x_1x_3, \sqrt{2}\,x_2x_3)^T,$$

where x_1, x_2, x_3 are the components of the input vector, $\vec{x}$. That is, $\vec{\phi}$ is a feature map which maps a vector into a higher-dimensional space.

Show that $k(\vec{x}, \vec{y}) = \vec{\phi}(\vec{x}) \cdot \vec{\phi}(\vec{y})$. That is, that k indeed computes the inner product of vectors in the higher-dimensional space.