DSC 140A - Midterm 01 Review

Problem 1.

Suppose $f(\vec{x})$ is a **convex** function of $\vec{x}$ and that the gradient of f at the point $\vec{x}^{(1)} = (0, 5)^T$ is $(-3, 5)^T$.

Let $\vec{x}^* = (x_1^*, x_2^*)$ be the minimizer of f; you can assume that $\vec{x}^* \neq \vec{x}^{(1)}$, which implies that $f(\vec{x}^*) < f(\vec{x}^{(1)})$. True or False: it must be the case that $x_2^* \geq 5$.

Problem 2.

Recall that a subgradient of the absolute loss is:

$$\begin{cases} \text{Aug}(\vec{x}), & \text{if } \text{Aug}(\vec{x}) \cdot \vec{w} - y > 0, \\ -\text{Aug}(\vec{x}), & \text{if} \, \text{Aug}(\vec{x}) \cdot \vec{w} - y < 0, \\ \vec{0}, & \text{otherwise.} \end{cases}$$

Suppose you are running subgradient descent to minimize the risk with respect to the absolute loss in order to train a function $H(x) = w_0 + w_1 x_1 + w_2 x_2$ on the following data set:

x_1	x_2	y
2	5	6
1	2	3

Suppose that the initial weight vector is $\vec{w} = (0, 0, 0)^T$ and that the learning rate $\eta = 1$. What will be the weight vector after one iteration of subgradient descent?

Problem 3.

Let $X = \{\vec{x}^{(i)}, y_i\}$ be a data set of n points where each $\vec{x}^{(i)} \in \mathbb{R}^d$.

Let $Z = \{\vec{z}^{(i)}, y_i\}$ be the data set obtained from the original by standardizing each feature. That is, if a matrix were created with the i-th row being $\vec{z}^{(i)}$, then the mean of each column would be 0, and the variance would be 1.

a) Suppose linear predictors H_1 and H_2 are fit on X and Z by minimizing the MSE, respectively.

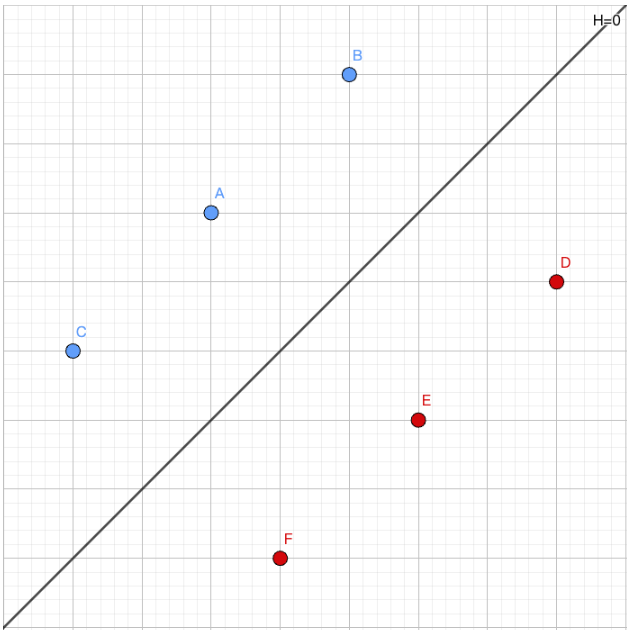
True or False: $H_1(\vec{x}^{(i)}) = H_2(\vec{z}^{(i)})$ for every $i = 1, \ldots, n$.

b) Suppose that X and Z are both linearly-separable. Suppose Hard-SVMs H_1 and H_2 are trained on X and Z, respectively.

True or False: $H_1(\vec{x}^{(i)}) = H_2(\vec{z}^{(i)})$ for every $i = 1, \ldots, n$.

Problem 4.

Consider the image below:



The blue points have label $+1$, and the red points have label -1. Suppose H is a linear prediction function, and when H is applied to the point A in the above image, $H(\vec{x}) = 4$. The black line in the middle of the image is the decision boundary $H = 0$.

You may assume that H is exactly in the middle of the points, and that all points are equidistant from the decision boundary.

a) What is the mean square loss of this prediction function, H?

b) True or false: there exists a linear prediction function H which has a mean square loss of zero on this data.

Problem 5.

a) True or false: Assume $f(x) = f_1(x) + f_2(x) + \ldots + f_k(x)$. If f is convex, then all $f_1, \ldots, f_k$ are convex.

b) True or false: Assume $f(x) = g(h(x))$. If f is convex, then both g and h are convex.

c) True or false: If f is convex, then $-f$ is non-convex.