DSC 140A - Discussion 04

Problem 1.

Using the definition, show that the function $f(\vec{w}) = a\vec{x} \cdot \vec{w} - b$ is convex as a function of $\vec{w}$, where $a, b \in \mathbb{R}$ and $\vec{x}, \vec{w} \in \mathbb{R}^d$.

Problem 2.

Let $f_1(\vec{w})$ and $f_2(\vec{w})$ be convex functions from $\mathbb{R}^d$ to $\mathbb{R}$. Define

$$f(\vec{w}) = \max\{f_1(\vec{w}), f_2(\vec{w})\}.$$

Show that $f(\vec{w})$ is convex.

Problem 3.

Recall that the Perceptron loss is:

$$L_{\text{perc}}(\vec{w}, \vec{x}, y) = \begin{cases} 0, & \text{if } \operatorname{sign}(\vec{w} \cdot \vec{x}) = y \text{ (correctly classified)}, \\ |\vec{w} \cdot \vec{x}|, & \text{if } \operatorname{sign}(\vec{w} \cdot \vec{x}) \neq y \text{ (misclassified)}. \end{cases}$$

Using the trick that $-y\,\vec{w} \cdot \vec{x} = |\vec{w} \cdot \vec{x}|$ in the case of misclassification, this can be be written in the equivalent form:

$$L_{\text{perc}}(\vec{w}, \vec{x}, y) = \max\{0, -y\,\vec{w} \cdot \vec{x}\}$$

Argue that the perceptron loss is convex as a function of $\vec{w}$.