DSC 140A - Math Check Worksheet

One of the things that can make machine learning theory challenging to learn is that it builds upon a lot of different math classes that you might have taken a long time ago – classes like multivariate calculus, linear algebra, and probability. In DSC 140A, we'll spend some time in lecture reviewing the background math, but we won't have time to cover everything in depth.

This worksheet is meant to be a way to give yourself a quick diagnostic check on your math background. **It's not graded, and it won't be turned in**. Instead, the idea is that you can use it to see what you remember and what you've forgotten, and to use the results to see what you might want to brush up on.

This worksheet is paired with a set of slides that cover the background math in more detail. These slides list a bunch of important "Facts" that you probably saw in a previous class, but may have forgotten. (Almost) every question below will use one or more of these facts. We'll also be using these facts regularly in the rest of the course.

You can find the slides at:

`http://dsc140a.com/materials/default/supplementary/math_review/slides.pdf`

Instructions:

Here's how we recommend using this worksheet:

1. Try each of the problems below without looking at the solutions.
2. Check the solutions to see if you got the right answer. You can find the solutions at

   `http://dsc140a.com/materials/default/supplementary/math_review/solution.pdf`.
3. If you got the wrong answer or just feel like you need more practice, look up the relevant part in the slides. Each problem below uses one or more important "Facts" that you probably learned somewhere in a previous class. The associated slides list these facts and explain them. If you're still unsure after taking a look at the slides, feel free to come to office hours (preferred) or ask on Campuswire!
4. Do a quick scan of the slides to see all of the "Facts", since some of them might not have been used in the problems below.

1 Summation Notation

Problem 1.

True or false: $\sum_{i=1}^{n} 6(x_i + 10) = (6\sum_{i=1}^{n} x_i) + 60n$

Problem 2.

How should we interpret $\sum_{i=1}^{n} x_i + y_i$?

◯ $\sum_{i=1}^{n}(x_i + y_i)$

◯ $(\sum_{i=1}^{n} x_i) + y_i$

2 Vectors

Problem 3.

Compute $(1, 4, 3)^T + (2, 0, 1)^T$.

Problem 4.

Compute $4(1, 4, 3)^T$.

Problem 5.

Compute $(1, 4, 3)^T \cdot (2, 0, 1)^T$. Here, $\cdot$ denotes the dot product.

Problem 6.

Two vectors $\vec{u}$ and $\vec{v}$ are orthogonal to one another (the angle between them is $90°$). What is $\vec{u} \cdot \vec{v}$?

Problem 7.

$\vec{u} = (1, 2, 3)^T$. What is the length of $\vec{u}$?

Problem 8.

Suppose $\vec{v} = (3, 3)^T$.

1. Find a unit vector $\vec{u}^{(1)}$ such that $\vec{u}^{(1)} \cdot \vec{v} = 0$.
2. Find a unit vector $\vec{u}^{(2)}$ such that $|\vec{u}^{(2)} \cdot \vec{v}|$ is maximized.

Problem 9.

Which of these is another expression for the norm of $\vec{u}$ (that is, $\|\vec{u}\|$)?

◯ $\vec{u} \cdot \vec{u}$

◯ $\sqrt{\vec{u}^2}$

◯ $\sqrt{\vec{u} \cdot \vec{u}}$

◯ $\vec{u}^2$

Problem 10.

Let $\vec{u}, \vec{v}, \vec{w}$ be vectors, and let α, β be scalars.

True or False: $\vec{u} \cdot (\alpha\vec{v} + \beta\vec{w}) = \alpha\vec{u} \cdot \vec{v} + \beta\vec{u} \cdot \vec{w}$.

3 Matrices

Problem 11.

Let

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{pmatrix},$$

and let $\vec{x} = (0, 1, 0, 2, 0)^T$.

What is $A\vec{x}$?

Problem 12.

Let A, B, C, X be matrices of appropriate dimensions. True or False: $X(AB + C)^T = XB^TA^T + XC^T$.

Problem 13.

Let A, B and C be matrices of appropriate dimensions.

True or False: $ABC = CBA$.

4 What type of object?

Problem 14.

Let $\vec{x} \in \mathbb{R}^d$ and let A be a $d \times d$ matrix. What type of object is $\vec{x}^T A\vec{x}$?

Problem 15.

Let $\vec{x}^{(1)}, \ldots, \vec{x}^{(n)}$ be d-dimensional vectors. What type of object is:

$$\frac{1}{n} \sum_{i=1}^{n} \vec{x}^{(i)} (\vec{x}^{(i)})^T$$